MATH 1910: Exam 1 Bonus Name: UUID:

Each question is open ended. There is no right answer and it will be graded based on effort alone. Feel free to use the textbook or material from other sources to investigate these questions. I also don't mind if you work together on it. For each problem you can get either 0 or 2 points added to your exam score. The only requirement to score 2 points on a single problem is that you show me that you made an effort to think about it. It won't matter if you don't understand or get something wrong. So you can get as much as 8 points added to your gross exam score. Since it was an 80 point exam this corresponds to a whole letter grade.

Due: October 5th

1. Recall the power rule: $\frac{d}{dx}x^n = nx^{n-1}$ for all postive integers n. More generally, it's true that $\frac{d^k}{dx^k}x^n = \frac{n!}{(n-k)!}x^{n-k}$ for all $k \leq n$. Here, $\frac{d^k}{dx^k}$, represents the kth derivative and $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$.

   Write down the first four derivatives of x^n. What pattern do you notice? Does it agree with the general rule I've given you for the kth derivative of x^n? What is $\frac{d^n}{dx^n}x^n$? (Hint: it's not d^{n-1} haha.) What do you think $\frac{d^k}{dx^k}x^n$ is for $k > n$?

2. Consider the function $f(x) = x^2$. How might we go about finding the number of tangent lines to this parabola which pass through a given point? What is the derivatie of x^2 at the point a? What is the equation of my tangent line at a? Draw the parabola as well as at least three separate tangent lines. Do you think a point above the parabola can be in any of the tangent lines? Why or why not? Suppose a point, (x, y), is in the tangent lines of the points $\left(a, a^2\right)$ and $\left(b, b^2\right)$ with $a \neq b$. Then

$$f'(a)\,x + a^2 - f'(a)\,a = y = f'(b)\,x + b^2 - f'(b)\,b.$$

   Solve for x in terms of a and b. Plug this representation of x into one of the equations for the tangent lines to get $y = ab$. Is $y < x^2$? Based on this where is this point in relation to the parabola?

3. Suppose two differentiable functions, f and g, differ by a constant, c. In symbols we express this as, $f(x) - g(x) = c$ for all real numbers x. This means that $f(x) = g(x) + c$. Differentiating both sides of this equality gives us $f'(x) = g'(x) + \frac{d}{dx}c = g'(x)$. (Remember that derivatives of constants are zero.) What does the result mean? Now, suppose I have

two functions, f and g, whose difference is dependent on x. That is $f(x) - g(x) = c(x)$ where c is a non-constant function. Now differentiate both sides of this equality. Could it be that $c'(x) = 0$? Why or why not? Can c' be constant? How is this result different from when f and g differ by a constant?

4. Let f be a function which is differentiable on the interval $[a, b]$. Then $\frac{f(b)-f(a)}{b-a}$ is not just the slope of the secant line between a and b but it is also the average value of f' on $[a, b]$. Why do you think that is? It might help to think of the derivative as the instantaneous rate of change. Suppose I want the average value of a general function f on $[a, b]$. Can I find a function g so that $g'(x) = f(x)$? How do I know when f is equal to the derivative of another function? If such a g exists how do I use it to find the average value of f?