MATH 1910: Exam 2 Bonus Name: UUID:

Each question will be graded based on effort alone. Feel free to use the textbook or material from other sources to investigate these questions. I also don't mind if you work together on it. For each problem you can get either 0 or 2 points added to your exam score. The only requirement to score 2 points on a single problem is that you show me that you made an effort to think about it. So you can get as much as 10 points added to your gross exam score. Since it was a 100 point exam this corresponds to a whole letter grade.

Due: November 18th

1. Find an antiderivative for $\tan(x)$. Try representing $\tan(x)$ as $\frac{\sin(x)}{\cos(x)}$. Then identify a function and it's derivative in that representation. You have two choices for the function: $\sin$ or $\cos$. Which choice allows you to do the chain rule backward?

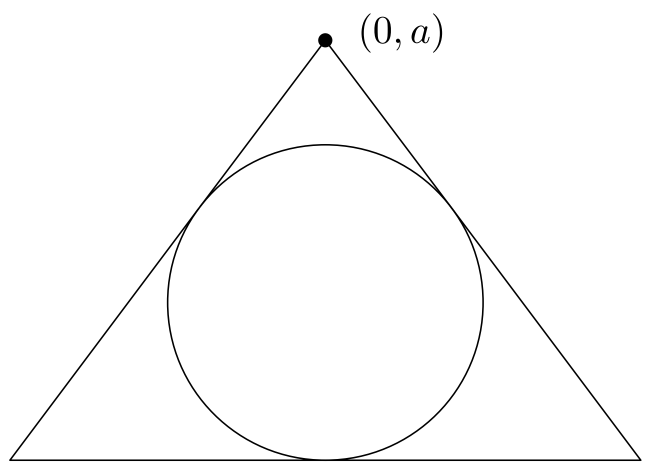
2. Draw a region whose area can be represented by

$$\int_{-1}^{1} \sqrt{1-x^2}\, dx - \int_{-1}^{1} |x| - 1\, dx.$$

Use basic geometry to calculate the area. Do the same for

$$\int_{-1}^{1} \sqrt{1-x^2}\, dx + \int_{-1}^{1} |x| - 1\, dx.$$

3. Consider the unit circle and an isosclese triangle that inscribes it so that the vertex joining the two equal edges is at $(0, a)$. Find the value of a that minimizes the lengths of the two sides.



4. Suppose you have an anologue clock whose battery is dying. At time t the radial speed of the minute hand is $m_{clock}(t) = \frac{\pi}{30}e^{-0.0001t} - 0.017$ radians/minute. How long will it take the clock to come to a complete stop? In other words, when is the radial speed zero? How many times will the clock be correct between $t = 0$ and then? Note that the speed of a correct clock would be $m_{real}(t) = \frac{\pi}{30}$ radians/minute. To find how often the broken clock is correct you need to find the positions of the broken clock and the correct clock. Assume that the broken clock is correct at $t = 0$. The broken clock is VISUALLY correct when the minute and hour hands are in the same position as the correct clock. We can express this mathematically by saying that the difference between the two positions is an integer multiple of $minutes * hours * \pi/30 = 60 * 12\pi/30 = 24\pi$. Find the difference between the two position functions at the time when the broken clock comes to a complete stop. Then divide by 24π. Round down to the nearest integer. This is the number of times the broken clock will be visually correct between $t = 0$ and the time it comes to a complete stop.

5. Minimize the distance between the two branches of $\frac{1}{x} - x$. The minimal distance between the two branches will occur at a point where the line through both points is perpindicular to the tangent lines at both points. The line perpindicular to the tangent line is called the normal line. Let's find the normal line for every point on each branch of our function. First calculate the derivative at each point. The normal line at a point x has slope $-1/f'(x)$. Then the normal line at a point a is $\frac{-1}{f'(a)}(x-a) + f(a)$. We need two points, with opposite sign, that have the exact same normal line. Suppose $a > 0 > b$ and that $\frac{-1}{f'(a)} = \frac{-1}{f'(b)}$. Solve for a in terms of b. You will get two values for a but only one of them is positive. Take the positive one since $a > 0$. Plug this value for a into the y-intercept of the normal line to f at a. Then set both y-intercepts equal to each other. You should have an equation only in b. Solve for b. Again you will get two values for b. This time you take the one that is negative. You should now have a value for b and a value for a. Use the distance formula to find the distance between $(a, f(a))$ and $(b, f(b))$. This is the minimal distance between the two branches.