MATH 1910: Exam 2 Name: UUID:

No calculators or phones or smartwatches are permitted. If you need extra paper or a pencil I will have some at the front. This is a 100 pt exam. There is a take home bonus that I will email to the class after the exam which is worth up to 10 pts.

1. Differentiate the following using differentiation rules. Label the following rules when you use them: product rule, quotient rule, chain rule, power rule. (10 pts each)

   (a) $\frac{d}{dx}\frac{x^2}{x^2+x+3}$

   (b) $\frac{d}{dx}\cos(x)\left(2x^2+x+1\right)$

2. Differentiate the following using differentiation rules. Label the following rules when you use them: product rule, quotient rule, chain rule, power rule. (10 pts each)

   (a) $\frac{d}{dx} \sin\left(x^2\right) e^{x^2}$

   (b) $\frac{d}{dx} 3^{x^2+x}$ (Hint: $\frac{d}{dx} b^x = b^x \ln b$ for any positive real number b. Including $b = 3$.)

3. Differentiate the following using logarithmic tricks. (10 pts each)

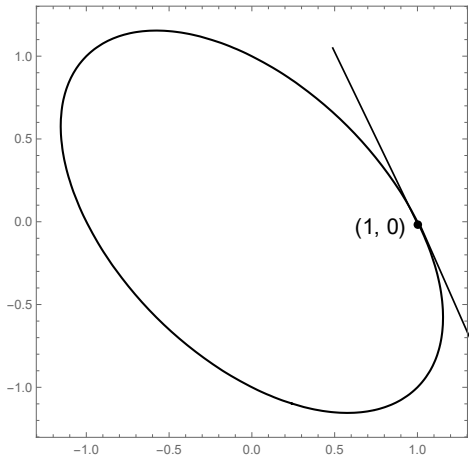
   (Hint: $\log_b(xy) = \log_b(x) + \log_b(y)$ and $\log_b(x^y) = y\log_b(x)$)

   (Hint: Try setting your function equal to y then taking ln of both sides. Apply the log rules above and then differentiate.)

   (a) $\frac{d}{dx} x^{\tan x}$

   (b) $\frac{d}{dx} x^{1/3}(x-1)^2\left(x^2+1\right)^{-1}$ (You may use the product rule here if you want. I won't count off if you do.)

4. For the equation, $x^2 + xy + y^2 = 1$, find y' by implicit differentiation. What is y' at the the point $(1, 0)$. The solutions to the equation, as well as the tangent line at that point, is illustrated below to help you check your answer. (20 pts)



5. The radius of a sphere is increasing at a rate of 2cm/second. How fast is the volume increasing when the radius is at 3cm. (Volume of a sphere $= \frac{4}{3}\pi r^3$) (15 pts)

6. Determine whether each statement is true or false. (1 pt each)

   (a) The derivative of a trigonometric function is a trigonometric function.

   (b) $\frac{d}{dx}10^x = x10^{x-1}$

   (c) There is an r between 0 and 1 so that the function $f(x) = x^r$ is differentiable at 0.

   (d) $\frac{d^{60}}{dx^{60}}\left(x^2 + x + 1\right)^{30} = 60!$

   (e) If f and $|f|$ are differentiable then the roots of f are also roots of f'.

This part is optional. Circle one:

Did you learn anything from this exam? If so, what?