Name:

No calculators or phones or smartwatches are permitted. If you need extra paper or a pencil I will have some at the front. This is a 100 pt exam. There is a take home bonus that I will email to the class after the exam which is worth up to 10 pts.

 Calculate the following limits using L'Hôpital's rule. Remember to check that it applies. (5 pts each)

(a) 
$$\lim_{x \to 0} \frac{\tan(3x)}{x}$$

(b) 
$$\lim_{x \to 1} \frac{x^{12} - 1}{x^3 - 1}$$
  
(c)  $\lim_{x \to 0^+} \frac{\ln(x)}{1/x}$ 

- 2. In this problem you will linearly approximate  $f(x) = \sqrt[3]{x+8}$  near 0 to estimate  $\sqrt[3]{9}$ . (5 pts each)
  - (a) Find the tangent line to f at 0.
  - (b) In order to approximate  $\sqrt[3]{9}$ , evaluate the tangent line at 1.

3. Find the max vertical distance between  $\sqrt{1-x^2}$  and  $x^2 - 1$  on the interval [-1, 1]. The graphs of the two functions on the given interval are illustrated below. The top one is  $\sqrt{1-x^2}$ . (10 pts)



- 4. Find antiderivatives for the following functions. (10 pts each)
  - (a)  $\frac{1}{x^2+1} + e^x$
  - (b)  $6x^2 + \frac{3}{2}x^{1/2}$
  - (c)  $2\cos(2x)$

- 5. For the function  $f(x) = x^5 5x^3$  find the following (5 pts each):
  - (a) the intervals where f is increasing and decreasing and mark them on graph I.
  - (b) the critical points of f and mark them on graph I.
  - (c) the intervals where f is concave up and down and mark them on graph II.
  - (d) the inflection points of f and mark them on graph II.
  - (e) the local max and mins of f and mark them on graph II.



- 6. Determine whether each statement is true or false. (2 pt each)
  - (a)  $\lim_{x \to 0} \frac{x}{e^x} = 1$
  - (b) Every continuous function has an antiderivative.
  - (c) If f is even and differentiable, then f' is odd.
  - (d) There is a function f such that f(1) = -2, f(3) = 0, and f'(x) < 0 for all x.
  - (e) If f'(c) = 0 then f has a local maximum or local minimum at c.

This part is optional. Circle one:



Did you learn anything from this exam? If so, what?